Option pricing for power prices with spikes

European power prices are very volatile and subject to spikes, particularly in German and Dutch markets. *Ronald Huisman* and *Cyriel de Jong* examine the impact of spikes on option prices by comparing prices from a standard mean-reverting model and a regime model that disentangles the spike process from the mean-reverting dynamics

For pricing of electricity derivatives, we cannot simply rely on models for financial and other commodity contracts. Electricity spot prices exhibit spikes, mean reversion, non-constant volatility and large seasonal variations because electricity is a commodity with limited storability and transportability – factors that strongly affect the behaviour of electricity spot and derivatives prices.

These peculiar characteristics have led researchers to develop special models for electricity prices. Although academic interest is growing, the number of papers addressing the specific valuation problems is relatively limited and only a few have been published to date. We recognise two different sets of power contract valuation approaches. The first consists of simultaneously modelling spot and forward contracts. This approach faces the difficulty that standard arbitrage principles cannot be applied to mapping spot prices to forwards and futures, since the concept of a convenience yield[1] is not applicable to a non-storable commodity. The second approach takes the market forward curve as given and is in contrast with the models above that derive the forward curve endogenously – that is, solely from the spot price process[2]. We follow the second approach.

Movements in electricity spot prices are not log-normally distributed and standard option-pricing formulas, based on normality assumptions, may yield incorrect outcomes. In order to derive an option-pricing methodology, we follow the approach of Clewlow and Strickland (1999) and Lucia and Schwartz (2002), but extend their mean-reverting framework with a new process for the spikes as a separate and independent regime. We therefore model power prices as a two-regime process: a normal regime for the mean-reverting process – which can be extended to include seasonality – and a spike regime. We show how an option price can be split into a mean-reverting and a spike component. Especially for out-of-the-money call options, this spike value represents the major component of total option value, as we show with an example based on Dutch Amsterdam Power Exchange (APX) spot prices.

The two-regime model for spot electricity prices

A standard mean-reverting specification is relatively successful in modelling commodities such as oil and natural gas. It is often applied to electricity markets, but with less success, because of the existence of spikes. Parameter calibration may lead to unrealistically high volatility, incorrect mean-reversion parameters and too-high price levels to which the spot prices would converge (see Huisman and Mahieu (2001)).

The most common approach is to model spikes using a stochastic jump model. Jump models allow for sudden extreme returns that lead to long-term shifts in price levels. Spikes are relatively short-lived and affect mean-reversion parameters, since the mean-reversion component is used to model the force that brings the spot price back to normal levels after a spike has occurred.

To incorporate the special characteristics of electricity spot prices, Huisman and Mahieu (2001) assume the dynamics of electricity prices can be described by different regimes. They propose a regime-switching model incorporating three regimes: a mean-reverting normal regime, an initial jump regime that models the process when prices suddenly increase or decrease, and a subsequent jump regime, that describes how prices are forced back to the stable regime. The main drawback of the Huisman and Mahieu (2001) model is that it does not allow for multiple consecutive jumps[3], which are common in electricity markets. Allowing for consecutive jumps is crucial for proper risk assessment and derivatives pricing.

In this paper, we propose a model with only two regimes: a stable, mean-reverting regime and a spike regime. It might seem unlikely that the omission of one regime gives the model the flexibility to capture consecutive jumps. However, we don't need a third regime to pull prices back to stable levels, because we assume that prices in the two regimes are independent from each other. Put differently, if there is a generator outage, for example, prices may be high for some time period, but once the generator is repaired, prices continue as normal. We believe this regime specification fits well with the structure of electricity markets. As a side effect of the independence of the two processes, we can split option values into a mean-reverting component and a spike component.

The two-regime framework

Following Lucia and Schwartz (2002), we model the natural logarithm price of spot electricity $s(t)$ as the sum of a deterministic component $f(t)$ and stochastic component $x(t)$. The first component, $f(.)$, accounts for predictable regularities, such as any periodic behaviour and trends, and is a deterministic function of time. The second component, $x(.)$, is the stochastic component, and we will refer to $x(.)$ as the spot price, but we must note that this actually is the spot price from which deterministic trends are removed.

In the two-regime framework, we assume that the spot price of electricity can be in one out of two regimes at each time period t. The regime reflects the normal behaviour of electricity prices and the second reflects the dynamics in the case of spikes. We assume that the deterministic trend $f(.)$ remains the same across both regimes[4]. The stochastic component on the other hand is different in each of the two regimes.

We refer to the spot price in the normal regime as $x(N,t)$ and to the

[1] *A commodity has a positive convenience yield if there is value in keeping the commodity in stock – for example, to keep the production process running at all times*

[2] *Examples of this approach are Clewlow and Strickland (1999), Bjerksund, Rasmussen and Stensland (2000) and Koekebakker and Ollmar (2001)*

[3] *Multiple consecutive jumps may theoretically be incorporated, but would require the estimation of a large number of switching probabilities*

[4] *See Hamilton (1989) as a reference on regime-switching models*

spike regime as $x(S,t)$. It is important to understand that the process is always in only one of the two regimes. We use this feature later to derive pricing formulas for options on spot prices.

Specification of the regimes

We follow Lucia and Schwartz (2002) and Clewlow and Strickland (1999) for deriving a process for the normal regime N that includes mean reversion. In this process, the parameter μ is the long-run equilibrium level for the natural logarithm of spot prices. The parameter α reflects the speed of convergence from the current to the equilibrium level (see table 1).

We assume that the behaviour of the natural logarithm of the spot price in the case of a spike can be modelled with a simple normal distribution whose mean and variance[5] are higher than those of the mean-reverting process.

Switching between regimes

At any point in time, the spot price $x(t)$ is either in regime N or S. To model the transition process over time, we use a Markov transition matrix. The Markov transition matrix Π is a 2x2 matrix with the elements $\pi(i,j)$ denoting the probability of going from regime i at time t to regime j in $t+1(i,j=N,S)$. Thus, $\pi(N,S)$ equals the probability that, while being in the normal process at time t, the process will be in the spike regime in period $t+1$.

We stated earlier that the two regimes are independent, which holds true for the prices in each regime. However, the above probability structure ensures there is a relationship between the two regimes in terms of the probability that they occur. For example, if we observe a spike today, then we know there is a greater probability of a spike tomorrow than when prices were normal. This is the type of relationship we observe in electricity markets.

Parameter estimation

The parameters of the two-regime model can be calibrated by applying maximum likelihood conditional on the regimes and assuming normally distributed error terms. Since the regime process $\lambda(t)$ is a latent process, the type of regime is not directly observable. The Kalman filtering methodology circumvents this problem and uses the prior and posterior beliefs to apply weights to each likelihood function[6].

Results from calibrating the model on Dutch APX data

As an illustration, we estimate the parameters of the two-regime model on the Dutch market. We use Dutch APX spot market data from January 2, 2001 to June 30, 2002 for base-load and peak day-ahead prices, totalling 545 observations for each price series. We define the deterministic part $f(t)$ such that it captures weekend effects, with a dummy for the Saturdays and a dummy for the Sundays and holidays. We found only very weak evidence of seasonality over the year, so no specification is included for it. The deterministic component $f(t)$ is estimated jointly with the stochastic model parameters.

[5] *Off-peak hours are characterised by negative spikes: the mean of the spikes becomes negative*

[6] *Harvey (1989) gives a description of this methodology that is the common method for estimating latent variables*

Table 1 lists the parameter estimates of the two-regime model, along with the results of calibrating a standard mean-reverting model and the Huisman and Mahieu (2001) three-regime model. The table indicates that both regime models improve the fit considerably compared with the mean-reverting specification.

Our two-regime model picks up, on average, 50% more spikes than the three-regime model of Huisman-Mahieu[7]. This is because their model requires an up-jump to be immediately followed by a down-jump and is thus more restrictive on jumps. The mean-reverting components of the two regime models are very similar in terms of mean-reversion speed and mean-reversion level. Moreover, the volatility parameters of the mean-reverting process in both regime models are considerably lower compared with the pure mean-reverting model. The mean-reverting volatility in our two-regime model is below that for the three-regime model; it transfers more erratic prices to the spike regime. The regime models indicate that the long-run average levels for baseload and peak spot prices are, respectively, €5 and €8 a megawatt hour (MWh) lower than under the mean-reverting model: a significant economic difference.

Our regime specification picks up spikes well: expected spikes are positive for base-load and peak, and have a much higher volatility than in the stable mean-reverting process. When prices were in the mean-reverting regime at t, a spike occurs with a probability of 10–13% for day $t+1$. The spikes in both regime models have an expected magnitude of between €53 and €58/MWh. Clearly, the spikes deviate considerably from the stable price levels.

[7] *In the Huisman-Mahieu model, we count the frequency of spikes as the sum of the up-spikes and down-spikes*

Table 1: **Parameter estimates**

		Base load	Peak load
Mean-reverting model	α	0.414	0.421
	μ	3.414	3.609
	σ	0.323	0.353
	Expected price	32.018	39.292
	Sunday dummy	–0.569	–0.613
	Saturday dummy	–0.231	–0.274
	Log likelihood	–0.288	–0.377
Huisman and Mahieu (2001) three-regime model	α	0.404	0.399
	μ_N	3.332	3.496
	σ_N	0.207	0.209
	Expected equilibrium price	28.589	33.727
	μ_S	0.590	0.583
	σ_S	0.559	0.570
	Expected spike	59.046	69.565
	Sunday dummy	–0.501	–0.528
	Saturday dummy	–0.230	–0.266
	$\pi(N,S)$	0.066	0.092
	Log likelihood	–0.109	–0.191
Two-regime model	α	0.356	0.243
	μ_N	3.289	3.433
	σ_N	0.157	0.123
	Expected equilibrium price	27.157	31.209
	μ_S	3.879	3.841
	σ_S	0.674	0.539
	Expected spike	57.760	53.826
	Sunday dummy	–0.468	–0.474
	Saturday dummy	–0.217	–0.252
	$\pi(N,S)$	0.084	0.139
	$\pi(S,N)$	0.473	0.385
	Log likelihood	–0.075	–0.137

Pricing European options on the underlying two-regime process

We now present an intuitive methodology for pricing option contracts on spot electricity that is assumed to follow a two-regime process, as described in the previous section. We present an approach for standard call and put options – deriving prices for caps, floors and swaptions is then straightforward.

Pricing options in the two-regime framework

In setting up the two-regime framework, we assumed that, at time t, the spot price is in one out of two independent regimes. We can therefore split up the option price in a component for the normal regime and one for the spike regime. For example, a European-style call option $c(X,\tau)$ with maturity τ and strike X has a 'fair' value that equals the expected pay-off of the option. At maturity, we know the process is in either the normal or the spike regime. Therefore, we can calculate the expected value of the option if we end up in the normal regime and the expected value in the spike regime.

Since both regime processes are log-normal, the well known Black (1976) formula is appropriate for the calculation. Then the actual option value equals the weighted sum of the two regime-dependent option values, where the weights equal the probability of ending up in each of the two regimes. These probabilities can be derived from the transition matrix P. The option value can thus be summarised as the probability-weighted average of two call options – one for the normal regime and one for the spike regime.

This breakdown of option values into two components clarifies the impact of spikes on option values and separates it from the normal price volatility. In a standard mean-reverting – or log-normal – framework, the extremely high volatility of spikes is averaged out against the moderate volatility of the normal regime. See, for example, the volatility estimates in table 1. This 'averaging-out' largely reduces the value of options, particularly longer-maturity out-of-the-money options, as we show below.

Making the expectations consistent with the current forward curve

Just as we split option values into a mean-reverting component and a spike component, we can split forward prices into two components. It is important to do this in order to align the model with market forward prices. We use the same approach as Clewlow and Strickland (1999) and Lucia and Schwartz (2002), who make the expectation of the spot price consistent with the forward curve. In fact, they adjust the long-run equilibrium level, μ, in a mean-reverting model to align with market expectations.

Once we have split forward prices into two components, we apply the same adjustment to our mean-reverting regime (see De Jong and Huisman (2002) for a detailed explanation). This adjustment avoids the tedious modelling and estimation of all seasonal influences and risk premiums. For example, we could not find historical evidence for seasonal variations in the Dutch APX data, but if the current forward curve suggests that prices in the winter will be higher than in the summer that information will be immediately incorporated in the price of options.

This is extremely useful: we don't want our derivatives to deviate from market prices because we have a different view on forward prices, but rather to build on market forward prices. On the other hand, we do have a different view on the spot price process, such as its volatility, level of mean reversion and magnitude of spikes, and need to incorporate this in the option valuation.

Table 2: **Prices for call options**

Maturity (days)	Forward price	Mean-reverting model Strike (€/MWh) 20.00	30.00	40.00	50.00	Two-regime model 20.00	30.00	40.00	50.00
				Panel A: Baseload options					
1	27.00	7.71	2.34	0.57	0.13	7.46	4.18	3.43	2.84
7	32.50	12.50	4.00	0.61	0.06	12.90	7.96	6.40	5.35
15	30.25	10.27	2.59	0.28	0.02	10.66	5.73	4.30	3.43
46	28.43	8.46	1.68	0.14	0.01	8.89	4.09	2.83	2.14
				Panel B: Peakload options					
1	38.00	18.13	9.77	4.52	1.90	18.01	11.72	10.08	8.68
7	41.75	21.73	11.87	4.14	0.86	21.79	14.47	11.31	9.18
15	40.75	20.71	10.87	3.41	0.59	20.77	13.32	10.11	8.05
46	41.25	21.12	11.30	3.68	0.67	21.18	13.73	10.50	8.41

Pricing results

In table 2, we compare prices for European call options between prices obtained from a standard mean-reverting model and our two-regime model. We analyse maturities of 1, 7, 15 and 46 days and strike prices of €20, €30, €40 and €50/MWh. We take the parameter estimates from table 1 and forward values for July 1, 2002 and assume an interest rate of 5%.

The option values that result from our two-regime spot price model largely deviate from options in a mean-reverting framework. The mean-reverting model suggests there is hardly any value in out-of-the-money options on spot prices that are more than five or 10 days ahead, as prices always revert back to a long-run level. In the mean-reverting model, volatilities decline exponentially to zero with increasing maturity.

The theoretical volatility of the base-load contract, for example, declines progressively from more than 13% for the day-ahead contract to less than 1% for the August contract (less than two months ahead), and the effect of the declining volatility on option values is strong. However, the regime-switch model takes into account that the trend might be towards such a long-run level, but prices might still deviate from it on individual days, due to spikes. The spikes imply that longer-term options also have substantial value.

Concluding remarks

We have presented a model for valuing options on electricity spot prices. It takes into account the two main features of electricity prices: strong mean reversion and occasional spikes. We obtained pricing results by disentangling the mean-reverting spot prices from the spikes, so that option values could be broken down into two components. We showed that it is crucial to include spikes in any option price formula, as they represent substantial value, especially for deep out-of-the-money options.

This result has important implications for capped electricity contracts. The costs of a maximum price – or cap – would be severely underestimated by the mean-reverting model. Caps equate to a series of call options and are often embedded in retail electricity contracts, where they form a bridge between fixed- and floating-price contracts.

Consider a contract where the end-user pays the daily base-load APX price on each day in July, but with a cap of €50/MWh. If we take the possibility of spikes into account, such a cap would cost around €3.43/MWh (based on an average maturity of 15 days), whereas a supplier would give it away almost for free (€0.02/MWh) if the wrong model were being used. EPRM

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